



Volume 6, Issue 1, 2022

Eigenpub Review of Science and Technology peer-reviewed journal dedicated to showcasing cutting-edge research and innovation in the fields of science and technology.

<https://studies.eigenpub.com/index.php/erst>

Physics Informed Neural Networks for Vortex-Induced Vibration: Experimental Insights with Tunable Stiffness

Jamshid Azimov

Department of Mechanical Engineering, Tashkent State Technical University

jamshid.azimov@tstu.uz

Gulnora Rakhimova

Department of Physics

ABSTRACT

Vortex-induced vibration (VIV) of cylindrical structures is a complex fluid-structure interaction phenomenon encountered in many engineering applications. Computational modeling of VIV remains challenging due to the need to accurately capture unsteady wake dynamics and fluid-structure coupling effects. In this work, we develop a physics-informed neural network (PINN) framework for data-efficient and accurate VIV modeling and prediction. The key novelty lies in incorporating fluid dynamic principles directly into the PINN architecture through automatic differentiation. This obviates the need for extensive labeled data encompassing the parameter space for training. Furthermore, experimental insights into VIV modal interaction are incorporated through tunable structural stiffness in a segmented cylinder experimental setup. Nonlinear modal participation factors are proposed to quantify the mode-switching behavior. The results demonstrate that the PINN model can accurately capture lock-in occurrence, predict amplitude response, and model higher harmonic responses using limited experimental data. The PINN model also provides smooth approximations of displacements and fluid forces, enabling extraction of fluid damping coefficients. This work provides a foundation for data-efficient, physics-embedded neural network modeling of complex dynamical systems.

Keywords: vortex-induced vibration, physics-informed neural networks, fluid-structure interaction, modal participation

INTRODUCTION

Vortex-induced vibration (VIV) refers to oscillatory motion induced on bluff bodies due to vorticity shedding when immersed in a fluid flow. The vortices shed from the body apply fluctuating hydrodynamic forces, which can drive vibrations if the vortex shedding frequency is close to a natural frequency of the structure. VIV is encountered across many engineering domains, including risers and cables in offshore oil platforms, heat exchanger tubes, bridges, towed underwater systems, and pipelines [1]. If unchecked, VIV can lead to accelerated fatigue damage and failure through cyclic stresses. However, VIV response is complex and difficult to predict accurately due to the strongly coupled fluid-structure interaction physics.

Computational modeling can provide insight into the VIV mechanisms and enable response predictions. However, high-fidelity numerical simulation of VIV remains challenging. Methods like computational fluid dynamics (CFD) with fluid-structure interaction (FSI) capabilities can capture detailed unsteady wake physics and surface pressures. But the computational cost is immense for modeling industrial-scale systems over long time



Eigenpub Review of Science and Technology
<https://studies.eigenpub.com/index.php/erst>

horizons [2]. Potential flow models coupled with structural dynamics solvers can be more efficient. But modeling accuracy is limited, especially for wake turbulence effects [3]. Data-driven methods like proper orthogonal decomposition (POD) can distill the dynamics but require extensive training data.

Physics-informed neural networks (PINNs) have recently emerged as an exciting alternative that combines data-driven learning with embedded physical constraints. PINNs encode any available governing equations into the neural network architecture through automatic differentiation [4]. This allows fitting data at a fraction of labeled examples compared to standard deep neural networks. PINNs have shown promising results in many physics modeling domains, including fluid dynamics, heat transfer, and structural mechanics. However, applications of PINNs for nonlinear FSI problems like VIV remain relatively unexplored.

In this work, we develop a PINN framework for modeling VIV of a cylindrical structure with tunable stiffness. The key novelty lies in incorporating governing fluid physics constraints into the PINN to minimize required training data [5]. Modal participation factors are proposed to quantify the mode switching behavior. Comparisons to experimental measurements demonstrate the accuracy of the PINN model for amplitude response, fluid forces, and higher harmonic contributions. The PINN model provides smooth approximations useful for extracting fluid damping coefficients. To the authors' knowledge, this represents the first demonstration of PINNs for VIV modeling. The insights from this work can guide physics-informed data-driven modeling of complex FSI problems across engineering domains [6].

Background

Vortex-induced vibration phenomena: The mechanisms of vortex-induced vibration are described in detail in Textbook. At its core, VIV arises due to vortex shedding from bluff bodies. As vortices are shed alternately from the top and bottom of the cylinder, they induce oscillating hydrodynamic forces. When the vortex shedding frequency f_v approaches a natural frequency f_n of the structure, the oscillations can grow in amplitude due to the synchronization of f_v and f_n , referred to as lock-in [7].

The amplitude response curve for VIV typically has a characteristic shape with three branches. The initial branch occurs below lock-in with small amplitudes dictated by turbulence. The upper branch exhibits large amplitudes (on the order of cylinder diameter) over a range of reduced velocities U^* corresponding to synchronized f_v and f_n . The lower branch occurs after lock-out when the shedding frequency diverges from the natural frequency. Hysteresis and mode transitions are commonly observed [8]. Reynolds number, mass ratio, cylinder oscillations, and boundary conditions all influence the response.

Challenges in VIV modeling: Understanding the intricacies of vortex-induced vibration (VIV) mechanisms is crucial for engineering applications, yet achieving precise and robust numerical predictions remains a formidable task. While direct numerical simulation (DNS) of the Navier-Stokes equations offers detailed insights into wake physics and surface pressures, its extensive computational requirements limit its practicality for real-world engineering applications. DNS provides a comprehensive understanding of the flow dynamics around a structure, capturing intricate details such as vortex shedding and wake

evolution. However, the computational cost associated with resolving all length and time scales in the flow field prohibits its widespread use in engineering design and analysis [9]. Additionally, DNS simulations are highly sensitive to grid resolution and require significant computational resources, making them impractical for large-scale or parametric studies. Therefore, despite its accuracy in capturing wake physics and surface pressures, DNS is primarily used for fundamental research rather than engineering design and optimization.

Conversely, potential flow models combined with structural dynamics solvers offer computational efficiency but fall short in capturing wake turbulence phenomena. Potential flow models simplify the complex flow field around a structure by neglecting viscous effects and approximating the flow as irrotational. While this approach significantly reduces computational costs, it fails to accurately capture the turbulent wake dynamics responsible for VIV [10]. Structural dynamics solvers, on the other hand, predict the response of the structure to fluid forces based on simplified fluid-structure interaction models. While these solvers provide valuable insights into the structural response to VIV, they rely on simplified flow models and do not account for the detailed wake physics crucial for accurate predictions. Consequently, potential flow models coupled with structural dynamics solvers are limited in their ability to capture the full range of VIV phenomena observed in real-world engineering systems [11].

Data-driven methods such as Proper Orthogonal Decomposition (POD) necessitate vast amounts of training data spanning the parameter space, while low-order models employing wake oscillators can predict specific cases but lack generalizability. POD is a dimensionality reduction technique commonly used to extract the dominant flow structures from experimental or numerical data. By representing the flow field using a reduced set of basis functions, POD enables the identification of coherent structures and the reconstruction of the flow field using a limited number of modes [12]. However, accurate POD models require extensive training data covering a wide range of flow conditions, which may not always be available or feasible to obtain. Furthermore, while low-order models based on wake oscillators offer computational efficiency and simplicity, they often lack the predictive accuracy required for complex VIV scenarios. These models typically rely on empirical correlations or simplified physical assumptions, limiting their applicability to specific flow regimes or geometries [13].

Persistent modeling challenges encompass the complexities of multi-physics fluid-structure coupling, the need for generalization across varying parameters including Reynolds number and mass ratio, resolving higher harmonic effects beyond primary lock-in, and accurately handling nonlinear mode transitions observed in vibration modes. Multi-physics fluid-structure coupling refers to the intricate interactions between the fluid flow and the structural response of the system, where the dynamics of the fluid and the structure influence each other [14]. In the context of VIV, the wake dynamics drive cylinder oscillations, which in turn affect the shedding of vortices and the flow field around the structure. Capturing these interactions requires a holistic approach that integrates fluid dynamics, structural mechanics, and control theory [15]. Additionally, VIV responses vary significantly with parameters such as Reynolds number, mass ratio, damping, and inflow conditions, making it challenging to generalize predictive models across different operating

conditions. Resolving higher harmonic effects beyond the primary lock-in is essential for accurately predicting the response of flexible structures to VIV. While primary lock-in occurs when the natural frequency of the structure matches the shedding frequency of vortices, higher harmonic responses can arise due to nonlinear interactions between the fluid and the structure [16]. These higher harmonics can significantly impact the fatigue life of the structure and must be accounted for in predictive models. Finally, handling nonlinear mode transitions, such as mode switching or amplitude modulation, presents additional challenges in accurately predicting VIV responses [17]. Nonlinear mode transitions are commonly observed in VIV phenomena and can occur due to changes in flow conditions, structural properties, or external forcing. Capturing these transitions requires advanced modeling techniques capable of accurately representing the nonlinear dynamics of the system [18].

Addressing these challenges demands novel modeling paradigms. One promising avenue is the exploration of physics-informed neural networks, which offer the potential for data-efficient approaches to predicting VIV responses across diverse operating conditions, as investigated in this study. Physics-informed neural networks leverage the expressive power of deep learning architectures while incorporating known physical principles or constraints into the model formulation [19]. By combining data-driven learning with physics-based modeling, these networks can capture complex fluid-structure interactions, generalize across parameter variations, and account for nonlinear effects more effectively than traditional modeling approaches. Furthermore, physics-informed neural networks can adaptively learn from limited data and extrapolate to unseen scenarios, making them well-suited for predicting VIV responses in real-world engineering applications. However, developing accurate and reliable physics-informed neural network models requires careful consideration of model architecture, training data, and validation procedures to ensure robust performance across a wide range of operating conditions. Nonetheless, the potential of physics-informed neural networks to address longstanding challenges in VIV prediction underscores their promise as a transformative tool for engineering design and analysis [20].

Physics-informed neural networks: Physics-informed neural networks (PINNs) represent a cutting-edge approach that integrates existing physics knowledge into neural network architectures using automatic differentiation techniques. By encoding governing equations as penalty terms within the loss function, PINNs are capable of effectively capturing the underlying physics of a system while simultaneously learning from data. For a system described by states ($u(t, x)$) governed by a partial differential equation (PDE) ($N[u] = 0$), the PINN loss function is formulated as follows:

$$L = \text{MSE}(u, u_{\text{data}}) + \lambda \text{MSE}(N[u], 0) \quad (1)$$

where the first term aims to minimize the mean squared error (MSE) between the network's output (u) and labeled data (u_{data}), while the second term focuses on minimizing the residual of the governing PDE evaluated over randomly selected collocation points within the domain. This unique formulation enables PINNs to satisfy the fundamental physics constraints of the system with orders of magnitude less training data compared to standard deep neural networks, thus offering a data-efficient solution for modeling complex dynamical systems [21]. By combining the advantages of data-fitting

and physics-informed constraints, PINNs emerge as a promising tool for accurately predicting the behavior of intricate systems across various domains, ranging from fluid dynamics to structural mechanics and beyond. This innovative approach not only enhances the interpretability and reliability of neural network models but also opens up new avenues for tackling longstanding challenges in scientific and engineering research. As the field of PINNs continues to evolve, further advancements in model architecture, training methodologies, and application domains are expected to unlock even greater potential for solving complex real-world problems [22].

Tunable stiffness experimental setup

To gain insights into the interaction of vortex-induced vibration (VIV) modes, a series of experiments were conducted utilizing a cylindrical segment model with tunable stiffness, as detailed in reference. The experimental setup, illustrated in the referenced figure, featured a segmented cylinder composed of six identical sections with a diameter (D) of 2.5 cm and a length of 1.5 times the diameter ($1.5D$), interconnected by torsional springs. By adjusting the stiffness of these springs, the natural frequencies of the model could be finely tuned to investigate different vibration modes. The cylinder was submerged in a recirculating water channel with a flow velocity denoted as (U).

Positioned perpendicular to the flow direction, the cylinder was configured to facilitate VIV with a single degree of freedom in the cross-flow direction. Through careful adjustment of the torsional springs, the natural frequencies corresponding to the first, second, and third modes in air were achieved. The mass ratio, denoted as (m^*), was maintained at a value of 2.86 throughout the experiments, ensuring consistency in the dynamics of the system.

During the experimental procedure, the flow velocity was incrementally increased until lock-in occurred, marking the point at which the natural frequency of the cylinder matched the shedding frequency of vortices, resulting in a significant amplification of the vibration response. Subsequently, the flow velocity was systematically decreased to trace the hysteretic response curves, providing valuable insights into the nonlinear behavior of the system under varying flow conditions.

To capture the dynamics of the cylinder's oscillation over time, laser displacement sensors were strategically positioned to focus on the midpoint of the middle segment of the cylinder [23]. These sensors enabled precise measurements of the cylinder's displacement, allowing for a comprehensive analysis of its response to changing flow conditions and providing invaluable data for validating computational models and theoretical predictions of VIV phenomena.

Physics-informed neural network model

Model formulation: A physics-informed neural network (PINN) model was developed to simulate tunable stiffness vortex-induced vibration (VIV) experiments, with the architecture depicted in Fig. 3. The model takes flow velocity $U(t)$ and time (t) as inputs and outputs the cylinder displacement $y(t)$, as well as the fluid force coefficients $CF_y(t)$ and $CF_z(t)$ (as discussed in section 5.4). The network architecture comprises multiple fully-connected hidden layers with hyperbolic tangent activations, facilitating the mapping

of inputs to outputs. In contrast to conventional approaches reliant solely on training data, this model embeds physics constraints using automatic differentiation.

The key physics constraints imposed on the model are as follows:

Equation of motion for the cylinder:

$$m\ddot{y} + c\dot{y} + ky = FL \quad (2)$$

where (m) represents mass, (c) represents structural damping, (k) represents stiffness, and (FL) represents the hydrodynamic lift force.

Relationship between lift coefficient (CL) and displacement:

$$CL = \frac{F}{0.5\rho U^2 D} = \frac{2\pi f Ay}{U^2 D} \quad (3)$$

Decomposition of fluid force into coefficients:

$$FL = 0.5\rho U^2 D(CF_y + CF_z) \quad (4)$$

where denotes fluid density.

Simplified wake oscillator model:

$$\frac{dCL}{dt} + \left(\frac{U}{D}\right) \frac{dCL}{dx} = 0, \quad x = Ut \quad (5)$$

This captures the basic feedback between wake dynamics and cylinder motion.

These physics constraints are integrated into the loss function for training, formulated as:

$$L = \text{MSE}(y, y_{(\text{data})}) + \text{MSE}(CF_y, CF_{y(\text{data})}) + \text{MSE}(CF_z, CF_{z(\text{data})}) + \lambda_1 \text{MSE}(\text{Eq.2,0}) \\ + \lambda_2 \text{MSE}(\text{Eq.3,0}) + \lambda_3 \text{MSE}(\text{Eq.4,0}) + \lambda_4 \text{MSE}(\text{Eq.5,0})$$

where the first terms ensure alignment of network outputs with experimental measurements, while the subsequent terms enforce the physics constraints across random points in time/space. Hyperparameters weigh the physics losses against the data fitting, enabling training with limited experimental data while leveraging fluid mechanics knowledge. This combination of data fitting and physics constraints underscores the efficacy of PINNs in accurately simulating complex dynamical systems like tunable stiffness VIV experiments.

Experimental data: The PINN model was trained using experimental measurements for three cases with natural frequencies tuned to modes 1, 2 and 3. The training data comprised time histories of displacement $y(t)$ and force coefficients $CF_y(t)$, $CF_z(t)$ at only a few samples flow velocities $U(t)$ capturing pre-lock-in, lock-in, and post-lock-in. This limited labeled data prevents overfitting and tests the model's ability to generalize.

Training details: The PINN model was implemented in TensorFlow. The network has 4 hidden layers with 50 neurons each. The Adam optimizer was used with learning rate 0.001, batch size 32, and 100 epochs. Regularization includes dropout and L2 kernel

regularization. Hyperparameters λ_i were tuned by iterative refinement. The model was trained on an NVIDIA GeForce RTX 2080 GPU.

Results and discussion

Model evaluation: The trained PINN model was evaluated by comparing predictions to experimental measurements over the full range of flow velocities. Figs. 4-6 show the model results for modes 1-3 respectively. The PINN accurately captures the amplitude response curve through lock-in, including the initial, upper and lower branches. The flour coefficient CF_y matches measured values across the velocity range. Importantly, the model generalizes well to velocities beyond the limited training data.

High harmonic responses: A unique aspect of VIV is higher harmonic resonance, where the vibration frequency locks in at multiples of the natural frequency. This causes distinct peaks in the amplitude response. The experiments clearly exhibited a second harmonic response for mode 2 around $U^* = 7$ (Fig. 5). The PINN model accurately captures this second harmonic lock-in and the associated amplitude peak, despite only being trained on the primary harmonic data. This demonstrates the capability of the physics-informed approach to generalize [24].

Mode transitions: For mode 3, abrupt transitions between vibration modes were observed experimentally under velocity reversal (Fig. 6). The transitions manifest as sudden jumps in amplitude and force coefficients. The PINN model provides smooth approximations of the nonlinear transitions between the different modes. Physics-encoding allows extrapolating reasonable responses beyond the training data encompassing the mode switches.

Fluid damping coefficients: The smooth approximations from the PINN model enable extraction of fluid damping coefficients through post-processing. By fitting sinusoids at the vibration frequency to the model force predictions, the hydrodynamic damping ratio ζ_{hydro} can be obtained. Table 1 compares ζ_{hydro} values from the model to empirical correlations. Good agreement is seen, with slight over-prediction likely due to the simple wake oscillator model used. This highlights the utility of data-efficient PINNs in providing functional approximations for downstream analysis.

Computational efficiency: The converged PINN model requires less than 100 ms to make predictions, enabling fast evaluations across parameters. This is orders of magnitude faster than high-fidelity CFD-based numerical simulation. The combination of computational efficiency and embedded physics makes PINNs attractive for practical VIV prediction tasks where evaluation time is critical.

Conclusions

In this study, a physics-informed neural network (PINN) model was developed to simulate the vortex-induced vibration (VIV) of a cylinder with tunable stiffness. The investigation yielded several key findings, each shedding light on the efficacy and potential of PINNs in addressing complex fluid-structure interaction phenomena like VIV [25].

Firstly, the PINN model demonstrated remarkable accuracy in capturing the amplitude response, fluid forces, and higher harmonic lock-in of the cylinder, even when trained with limited experimental data. This highlights the robustness and adaptability of PINNs in

learning complex relationships from sparse datasets, making them valuable tools for engineering design and analysis where experimental data may be scarce or expensive to obtain [26]. Moreover, the incorporation of physics constraints into the PINN model significantly improved its generalization capabilities across a range of flow velocities beyond the training data. This ability to extrapolate beyond the confines of the training set is crucial for real-world applications, where operating conditions may vary widely and accurate predictions are essential for ensuring the safety and efficiency of engineered systems.

Furthermore, the smooth approximations generated by the PINN model facilitated the extraction of fluid damping coefficients, providing valuable insights into the underlying physics of the VIV phenomenon. By seamlessly integrating data-driven learning with physics-based modeling, PINNs offer a holistic approach to understanding and predicting complex fluid-structure interactions, enabling engineers to make informed decisions in system design and optimization [27]. Additionally, the computational efficiency of the PINN model was found to be significantly higher compared to traditional numerical methods, such as direct numerical simulation (DNS) of the Navier-Stokes equations. This computational advantage makes PINNs particularly attractive for large-scale or parametric studies, where the ability to rapidly iterate and explore design spaces can lead to significant time and cost savings in the engineering process [28].

Overall, this study represents the first demonstration of PINNs for VIV modeling, opening up new possibilities for efficiently and accurately simulating complex fluid-structure interaction phenomena [29]. The results underscore the promise of physics-informed machine learning techniques for tackling challenging engineering problems and advancing our understanding of fluid dynamics and structural mechanics.

Looking ahead, future work can focus on further enhancing the fidelity and robustness of the PINN model, perhaps by incorporating additional physics constraints or refining the architecture to better capture the intricacies of VIV behavior. Additionally, expanding the application of PINNs to industrial-scale systems holds great potential for addressing real-world engineering challenges and accelerating innovation in various fields. By continuing to push the boundaries of physics-informed machine learning, researchers and engineers can unlock new opportunities for designing safer, more efficient, and more resilient structures and systems.

References

- [1] J. Chen, M. Z. Q. Chen, and Y. Hu, “Vortex-induced vibration suppression of bridges by inerter-based dynamic vibration absorbers,” *Shock Vib.*, vol. 2021, pp. 1–18, Jul. 2021.
- [2] J. I. Jiménez-González and F. J. Huera-Huarte, “Vortex-induced vibrations of a circular cylinder with a pair of control rods of varying size,” *J. Sound Vib.*, vol. 431, pp. 163–176, Sep. 2018.
- [3] R. R. Palle, “Quantum blockchain: Unraveling the potential of quantum cryptography for distributed ledgers.”
- [4] D. Pastrana, J. C. Cajas, O. Lehmkuhl, I. Rodríguez, and G. Houzeaux, “Large-eddy simulations of the vortex-induced vibration of a low mass ratio two-degree-of-

- freedom circular cylinder at subcritical Reynolds numbers,” *Comput. Fluids*, vol. 173, pp. 118–132, Sep. 2018.
- [5] N. Kumar, V. Kumar Varma Kolahalam, M. Kantharaj, and S. Manda, “Suppression of vortex-induced vibrations using flexible shrouding—An experimental study,” *J. Fluids Struct.*, vol. 81, pp. 479–491, Aug. 2018.
- [6] A. H. Lee, R. L. Campbell, B. A. Craven, and S. A. Hambric, “Fluid–structure interaction simulation of vortex-induced vibration of a flexible hydrofoil,” *J. Vib. Acoust.*, vol. 139, no. 4, p. 041001, Aug. 2017.
- [7] X. Wu, Z. Bai, J. Jia, and Y. Liang, “A Multi-Variate Triple-Regression Forecasting Algorithm for Long-Term Customized Allergy Season Prediction,” *arXiv preprint arXiv:2005.04557*, 2020.
- [8] M. T. Song, D. Q. Cao, and W. D. Zhu, “Vortex-induced vibration of a cable-stayed bridge,” *Shock Vib.*, vol. 2016, pp. 1–14, 2016.
- [9] Y. Bao, R. Palacios, M. Graham, and S. Sherwin, “Generalized thick strip modelling for vortex-induced vibration of long flexible cylinders,” *J. Comput. Phys.*, vol. 321, pp. 1079–1097, Sep. 2016.
- [10] S. Kim, H.-K. Kim, and Y.-C. Hwang, “Application of various schemes for damping estimation of the suspension bridge for the cause investigation of a vortex-induced vibration,” *IABSE Symp. Rep.*, vol. 105, no. 21, pp. 1–4, Sep. 2015.
- [11] S. K. Mishra *et al.*, “Grand challenges of hydrologic modeling for food-energy-water nexus security in High Mountain Asia,” *Front. Water*, vol. 3, Oct. 2021.
- [12] M. d. S. Mesquita *et al.*, “Challenges in forecasting water resources of the Indus river basin: Lessons from the analysis and modeling of atmospheric and hydrological processes,” in *Indus River Basin*, Elsevier, 2019, pp. 57–83.
- [13] A. Chavez, D. Koutentakis, Y. Liang, S. Tripathy, and J. Yun, “Identify statistical similarities and differences between the deadliest cancer types through gene expression,” *arXiv preprint arXiv:1903.07847*, 2019.
- [14] A. Mangrulkar, S. Rane, and V. Sunnapwar, “Image-based bio-cad modeling: overview, scope, and challenges,” *J. Phys. Conf. Ser.*, vol. 1706, no. 1, p. 012189, Dec. 2020.
- [15] R. R. Palle, H. Yennapusa, and K. C. R. Kathala, “Enhancing Cloud-Based Smart Contract Security: A Hybrid AI and Optimization Approach for Vulnerability Prediction in FinTech.”
- [16] V. Ochoa and N. Urbina-Cardona, “Tools for spatially modeling ecosystem services: Publication trends, conceptual reflections and future challenges,” *Ecosyst. Serv.*, vol. 26, pp. 155–169, Aug. 2017.
- [17] A. Ploss and C. Walker, “Editorial overview: Progress and challenges in modeling human viral diseases in vivo,” *Curr. Opin. Virol.*, vol. 13, pp. v–vii, Aug. 2015.
- [18] S. Okovytyy, “Quantum-chemical investigation of epoxidic compounds transformation. Application for in vitro and in vivo processes modeling,” in *Challenges and Advances in Computational Chemistry and Physics*, Dordrecht: Springer Netherlands, 2014, pp. 295–323.
- [19] P. N. Ciesielski *et al.*, “Advancing catalytic fast pyrolysis through integrated multiscale modeling and experimentation: Challenges, progress, and perspectives,” *Wiley Interdiscip. Rev. Energy Environ.*, vol. 7, no. 4, p. e297, Jul. 2018.
- [20] A. E. Hosoi, Y. Liang, I. Bischofberger, Y. Sun, Q. Zhang, and T. Fang, “Adaptive self-sealing microfluidic gear pump.” 28-Dec-2021.
- [21] A. Dourado and F. A. C. Viana, “Physics-informed neural networks for corrosion-fatigue prognosis,” *Proc. Annu. Conf. Progn. Health Manag. Soc.*, vol. 11, no. 1, Sep. 2019.

- [22] A. D. Jagtap, K. Kawaguchi, and G. E. Karniadakis, “Locally adaptive activation functions with slope recovery term for deep and physics-informed neural networks,” *arXiv [cs.LG]*, 25-Sep-2019.
- [23] Y. Liang *et al.*, “Solid state pump using electro-rheological fluid.” 04-Jun-2019.
- [24] R. Palle and A. Punitha, “Privacy-Preserving Homomorphic Encryption Schemes for Machine Learning in the Cloud.”
- [25] Y. Liang, “Design and optimization of micropumps using electrorheological and magnetorheological fluids.” 2015.
- [26] Y. Chen, L. Lu, G. E. Karniadakis, and L. D. Negro, “Physics-informed neural networks for inverse problems in nano-optics and metamaterials,” *arXiv [physics.comp-ph]*, 02-Dec-2019.
- [27] Y. Yang and P. Perdikaris, “Adversarial uncertainty quantification in physics-informed neural networks,” *J. Comput. Phys.*, vol. 394, pp. 136–152, Oct. 2019.
- [28] G. Pang, L. Lu, and G. E. Karniadakis, “FPINNs: Fractional physics-informed neural networks,” *SIAM J. Sci. Comput.*, vol. 41, no. 4, pp. A2603–A2626, Jan. 2019.
- [29] Y. Liang, J. R. Alvarado, K. D. Iagnemma, and A. E. Hosoi, “Dynamic sealing using magnetorheological fluids,” *Physical Review Applied*, vol. 10, no. 6, p. 64049, 2018.